

# Computer Erosion Modeling Considering Soil Hydraulic Conductivity

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## Abstract

Improved computer models of erosion have been developed, considering soil hydraulic conductivity. The models deal with erosion of levees, dams and embankments due to overtopping. The simulations trace the formation of rills and gullies, beginning with initial overtopping and continuing to final breaching. Physical models performed at 1-g and high g using a geotechnical centrifuge have been used to calibrate the models. Previous models did not consider soil hydraulic conductivity, and although results were quite good for the formation of rills and gullies and sediment quantities, breach times were underestimated. Essentially the water flow was treated as if passing over a solid surface, not entering the soil, and the total water flow was available for erosion. Thus, breach times were underestimated. Soil erodibility parameters had to be adjusted in order to achieve good agreement with breach times. The new models developed consider soil hydraulic conductivity, and produce good agreement

with the performance of the physical modeling, including breach times and the use of proper soil erodibility parameters.

**Keywords:** Erosion, Levees, Computer Simulations, Physical Modeling

## 1. Introduction

Flood disaster is caused by levee breach due to high river flow and consequences as huge damage occurs in the inhabitants inside the levees. It primarily occurs due to overtopping of the levees. However other major levee failure mechanisms initiate with seepage. In all mechanisms, the erosive processes can eventually lead to breaching of the levee and catastrophic damage on the adjacent flood plain. A reliable prediction of the flood process, especially in a complex terrain is necessary for emergency plans for levee or dam breaches. Islam and Tsujimoto [1] attempted to recognize the process of levee breaching and inundation through

numerical simulation. As for simulation scheme, schematic model area was considered with main channel, levee and floodplain composed of same materials. The channel and flow parameters were selected in conformity with the typical model field of Bangladesh as well as Japan. A two-dimensional numerical model for flood flow and morphology was utilized. Based on the calculated results, evolution process of levee breach and inundation of water and sedimentation in the floodplain were investigated. Levee breach was considered to initiate in the middle of the levee with crest opening. The results showed that levee breach with steeper river bed slope has high risk of flood disasters in the floodplain. Therefore, River Management Authority should consider proper designing of levee to protect levee failure and adequate dredging techniques is necessary to maintain normal river flow capacity, which decrease the risk of levee breach and furthermore to minimize flood disasters suffering the inhabitants on unexpected breach happening.

Zinke et al. [2] used a three-dimensional numerical model to compute the water flow and the sediment deposition for a 100-year flood in the delta of Lake Qyeren in Norway. The model solved the Reynolds-averaged Navier-Stokes (RANS) equations with the k- $\epsilon$  turbulence model on a three-dimensional unstructured grid. The sediment transport was computed solving the convection-diffusion equation for sediment concentration using an empirical formula for the bed pickup rate. The results were tested against sedimentation data obtained from field measurements in a qualitative way. A series of sensitivity tests showed the important role of uncertainties in the bed roughness parameter, the incoming sediment concentration, the grain size distribution on the channel bed, and the vegetation parameters. The presence of vegetation strongly affected the position and intensity of the sediment depositions, and its proper parameterization plays a key role for the modeling.

Present criteria for acceptable grass covered levee overtopping are based on average overtopping values but do not include the effect of overtopping duration. Dean et al. [3] applied experimental steady state results for acceptable overtopping to the case of intermittent wave overtopping. Laboratory results consisting of velocities and durations for acceptable land side levee erosion due to steady flows were examined to determine the physical basis for the erosion. Three bases were examined: (1) velocity above a threshold value, (2) shear stress above a threshold value, and (3) work above a threshold value. The governing equations for flow down the land side of a levee established that the flows near the land side levee toe will be supercritical. Wave run-up was considered to be Rayleigh distributed with the run-up above the levee crest serving as a surrogate for overtopping.

Griffis [4] addressed the overall design of the engineering works that protect the City of New Orleans from major storm events. The history of the drainage of the city was briefly described, and the natural and engineered flood-protection structures and strategy for major hurricanes were discussed. This was followed by an analysis of the failures of the natural system, as well as failures of the levees, floodwalls, pumping stations, and the absence of flood gates on the canals. He eventually concluded that the "design storm" specified by congressionally authorized projects deserve reconsideration, and that engineering design reviews should be reorganized and implemented.

Wan and Fell [5] describe the development of two erosion rate tests, the Hole Erosion Test (HET) and Soil Erosion Test (SET), which measure soil erodibility. Using an Erosion Function Apparatus (EFA), Briaud et al [6] investigated the erodibility of several different types of soil. The soils were classified into different categories of erodibility based on degree of compaction, erosion rate, water velocity and hydraulic shear stress. Xu and

Zhang [7] found that in addition to soil type, the degree of compaction plays an important role in erodibility of embankments. The erosion resistance increases with compaction effort, particularly with fine soils. Bryan and Rockwell [8] studied agricultural sites near Toronto, Canada and found that significant rill incision typically occurred in early spring, immediately following snowmelt. However, this study relates levees or earth dams that are adjacent to water bodies and are saturated or can become saturated rapidly. Rills and gullies will form in areas of depression, or in areas where the soil does not have enough cohesion or shear strength to resist the hydraulic stresses from the flowing water. Factors affecting rill characteristics include the stress caused by the flow, roughness of the soil surface, slope gradient and soil erodibility [9]. However, Govers, et al. [10] stated that erodibility within a rill may vary with its depth, which can decrease the erosion process in granular soils, as a result of a reduced slope gradient. If a more erodible soil underlies the surface soil, however, the erosion rate in a rill or gully will actually be accelerated.

Post Hurricane Katrina field surveys showed that in general, rolled compacted clay filled levees performed well with minor erosion occurring when overtopped, whereas hydraulic filled levees with significant amounts of silt and sand performed poorly. Using good clayey material often required long haul distances that slowed construction progress. So nearby granular material was often used instead to construct the levees [11]. In cohesive embankments, breaching occurs as a result of head cutting, whereas in granular embankments, surface slips occur rapidly due to seepage on the downstream slope [7].

Experience resulting from Hurricane Katrina has shown that land side levee erosion due to wave overtopping can significantly limit levee performance and survival [12]. Erosion is a time dependent process such that a levee can withstand various overtopping magnitudes for different durations. Although the specific interest may be in designing the levee for survival during a particular storm (e.g. a 100 year event), there is also interest in the erosional potential during storms that will cause greater overtopping. Flor et al. [13] tested the relative importance of geologic, geomorphic, and other physical factors that have led to levee failures through the past century along the Mississippi River and presented some results that could potentially assist engineers and decision-makers in choosing appropriate locations and designs for levees. Kamalzare et al. [14] used SPH to prepare a model to simulate the water movement and the interaction of the water particles with soil particles. A large number of physical laboratory experiments were conducted to find the necessary coefficients required for the computer model. Yu et al., [15] carried out numerical simulations of levee or dam breach flow, often with constant flow parameters and in relatively simple channels rather than in natural rivers with complex boundaries using 2-D finite element models. The good performance of the model was demonstrated by comparisons of breaching with the theoretical solution of an idealized dam-break flow over a frictionless flat rectangular channel. The model was applied to simulate the flood propagation under complex boundary conditions. The unsteady flood process in a river and in the dry floodplain with a complex bed terrain was also simulated simultaneously. Kamalzare et al. [16] studied the levee's erosion due to overtopping and prepared a computer simulation that would model the overtopping. The model was validated with physical models for different levee sizes and different soil types [17]. A new visualization method was also introduced to improve the physical erosion measurements, and consequently increase the validation precision [18]. Xiao et al., [19] applied a numerical wave model based on the incompressible Reynolds equations and k- $\epsilon$  equations to estimate the impact of overtopping on levees

during storm surge. The free surface locations were represented by a volume of fluid function (VOF). The model was satisfactorily tested for an empirical equation of overflow discharge at a vertical seawall and experimental data of overtopping discharge at a sloping seawall. The validated model was used to simulate wave overtopping of the levee system during storm surge of Hurricane Katrina. The time history of wave profiles and velocity magnitude fields in the vicinity of the levees were demonstrated and analyzed.

As computer capabilities progress in representing hurricane induced storm surges, there is a need to improve understanding of the overtopping erosion potential and to provide associated guidance for more rational design. Although much work has been done to simulate erosion, very little of the results have been validated. A primary objective of this research was validation of the computer simulation by laboratory experimentation. Therefore in this paper, laboratory tests with different soils have been performed to improve the computer simulations of levee erosion. Previous tests have been performed using different mixtures of two soils and the effects of different percentages of clay have been investigated previously [20]. The emphasis of this paper was to investigate the effect of water flow on the erosion. Therefore, all the tests were performed on one mixture of soil (25% clay, 75% sand) and water was added using various water flow rates. To better evaluate the effects of water flow on real levees, some centrifuge tests have also been performed which simulate full scale prototype levees and embankments.

## 2. Materials and test procedures

A mixture of two soils have been used in the tests represented herein, a clay soil (Kaolinite Clay) and a granular soil (Nevada 120 Sand). Tests were performed on mixtures of 25% clay and 75% sand, which is almost prevalent in many levees around the world. Maximum dry density and optimum water content of the sand and clay were  $16.4\text{kN/m}^3$  and 11% for the sand and  $12.8\text{kN/m}^3$  and 29% for the clay respectively. Table 1 lists the physical characteristics of the mixed soil, while Fig. 1 shows grain size distribution curves of the pure sand and the mixed soil. The mixed soil is classified as SC according to the Unified Soil Classification System (USCS). The maximum dry unit weight for the soil sample was  $15.4\text{kN/m}^3$ . Samples were prepared to achieve a relative density of 90% of the maximum dry density ( $13.9\text{kN/m}^3$ ) and used the optimum water content (10%) which has been calculated according to AASHTO T99-70 / ASTM D698-70 (A-method).

Table 1: Soil Characteristics

Property	Mixed Soil
D <sub>10</sub> (mm)	0.074
D <sub>30</sub> (mm)	0.11
D <sub>60</sub> (mm)	0.19
CU'	2.57
CC'	0.86
Liquid Limit	17
Plastic Limit	11
Permeability (cm/s)	10e-5
USCS symbol	SC

\*CU: Coefficient of uniformity; CC: Coefficient of curvature

The models used in this research were constructed in an aluminum box having a wall thickness of 0.0254m and interior dimensions of 0.87m L x 0.39m W x 0.36m H. The geometry of the model levee was determined similar to conventional real levees before construction of the model began. The dimensions were marked on the sides of the model box at the proper angles to ensure that the model levee was constructed to the desired specified geometry. Some other tests were also performed in smaller boxes to find the effect of dimensions of levee on the results. The compaction of soil was conducted manually by using a plastic hammer to hit the steel plate, which was placed on top of the soil until reaching the target unit weight.

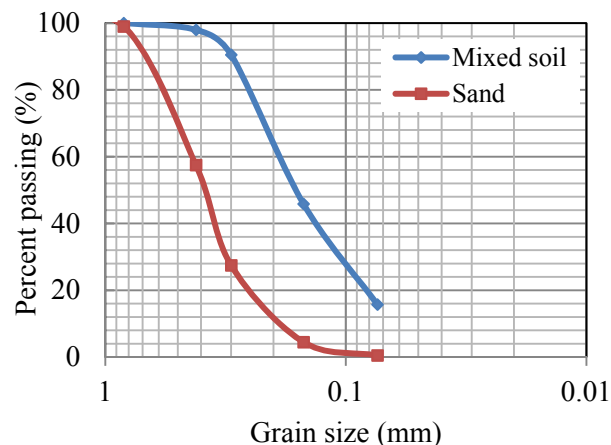


Fig. 1. Grain size distribution curves of soils

Small-scale erosion on earthen embankments is being studied, modeled and simulated, with respect to the formation of rills and gullies. Validation of the simulation was one the primary focus in this research. Therefore scaled-down model levees are used to perform erosion experiments at 1-g and at higher levels of g in a geotechnical centrifuge. The results of experiments to date are presented in the following sections. Different water flows were used and complex geometries and boundary conditions utilized to quantitatively assess the effects of differing conditions. The physical models serve as the basis for developing accurate, digital simulations of the embankment erosion processes. It should be mentioned that the erosion processes described in this paper refer to hydraulic erosion. The time elapsed from initiation of initial rill erosion began at the crest of the landside slope to the time the eroded channel reached the crest on the waterside slope ( $t_{breach}$ ) was measured during the tests. Photographs and videos were taken before, during and after each test.

In order to do some large scale measurements centrifuge tests were performed at 20g's (Fig. 2). In high g tests water will become heavier and erosion will occur much faster than 1-g. Therefore a high speed camera was used to take pictures and record videos during the tests in centrifuge. The camera was a V5 High Speed CMOS Camera with full image resolution 1024x1024 pixel array capability at 60,000 pps. Two other cameras were also recording videos from different angles. These videos and pictures were being used to evaluate the results of digital simulations and computer predictions.



Fig. 2. RPI 150 g-ton geotechnical centrifuge

Erosion simulation system is based on theories of Smoothed Particle Hydrodynamics (SPH) [21]. Both the water and the levee are discretized by particles, and the behaviour of fluid is modeled by the Navier-Stokes equations. In each of the simulations, approximately 450,000 and 2,500,000 particles are used to represent the water and the soil, respectively ([22]). In simulations, the erosion rate, “ $z$ ”, (mm/hr) is modeled by using Eq. 1:

$$Z = \begin{cases} 0 & \text{when } \tau \leq \tau_c \\ a \times \tau + 0.1 & \text{when } \tau > \tau_c \end{cases} \quad (1)$$

Where  $\tau$  is the hydraulic shear stress (Pa) and  $\tau_c$  is the critical shear stress. Since the values of  $a$  and  $\tau_c$  are different for different materials, their values have to be determined for each material used in physical experiments. In the authors' previous experiments, pure sand and sand-clay mixtures (85% sand and 15% clay) had been used as levees material. In the previous simulations, the value for  $a$  was estimated to be 187 and 93 for pure sand and sand-clay mixtures respectively, and the value for  $\tau_c$  was estimated to be 2.0 and 3.0. A series of simulations on those two materials have been run, as well as some imaginary materials whose erodibility lies between the erodibility of those two materials [23]. In order to determine the values of the parameters for the material of current experiments, a comparison between the results of previous simulations and the results of current physical experiments have been done. Water flow rate, geometry of the levee surface, and erodibility of the soil were identified as three major components in the formation of channels during erosion simulation. A total of 27 computer simulations have been run, one for each possible combination of three different flow rates, levee down-slope angles, and erodibility values. For flow rates, values of 8, 11, and 14 mL/s, relatively fast but not unrealistic rates were chosen. For erodibility values, 137, 159, and 187 alpha-values, representing the range from sand-clay mixture made up of approximately 10% clay to pure sand were chosen. Finally, for levee slope, dry-side slopes of 4:1, 5:1, and 6:1, ranges found in real levee design were chosen. For each simulation result, the time to breach was visually determined, as been identified by the Dam-Break Flood Forecasting Model.

### 3. Results and discussions

Several levees were built in an aluminum box and the effect of over topping on the levee failure was investigated. It is clear that the bigger the levee built, the more accurate and reliable results can be found. However it would be difficult and time consuming to build several big levees. Therefore to find the most efficient size of the levee that lead to precise results, a number of tests were performed in three different boxes with different dimensions, the small box (38.0cm×37.5cm), the medium box (87.5cm×39.5cm) and the large box (91.0cm×61.0cm). The shapes of the rills and the time that takes for levee to fail was compared for levees built in boxes. It was found that among all the levees, the one in the medium box had the most efficient size that could be built in a reasonable time and lead to accurate results. Fig. 3 sketches the dimensions of the medium box and the levee built inside.

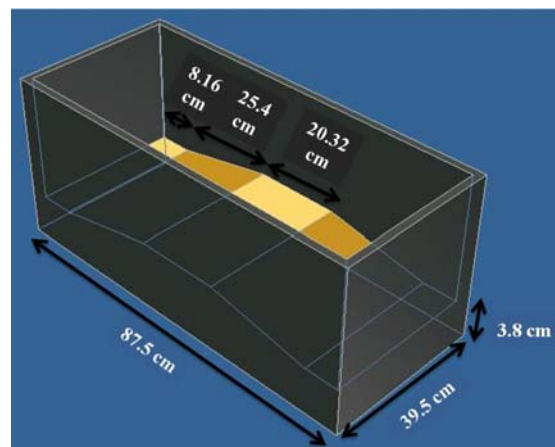


Fig. 3. Dimensions of the modeled levee in the medium box

Several 1-g tests with different water flows were conducted and different characteristics of the rills such as their shape and depth were recorded. This information was used to calibrate the computer simulations. To have a better validation, four different times were defined and measured during the tests:  $t_{\text{elevation}}$ , the time duration for water (at a specific flow rate) to fill the upstream and reach the elevation of the crown,  $t_{\text{cross crown}}$ , the time duration for water to cross the crown of the levee,  $t_{\text{rill}}$ , the time elapsed from initial rill formation at the crest of the landside slope to the time the rill reached the toe of the slope, and  $t_{\text{breach}}$ , the time elapsed from initiation of initial rill erosion began at the crest of the landside slope to the time the eroded channel reached the crest on the waterside slope.

However to study the effects of over topping on bigger models, centrifuge tests were conducted. In centrifuge, time and dimensions scale by  $g$ . Therefore the experiment using medium box (Fig. 3) with water flow equal to 0.56 lit/min that took 5 minutes to fail, resembles a 17.50m long levee with 1.78m height and 7.90m width that fails in 100 minutes in real life. Fig. 4 shows the levee after failure in centrifuge test.

Times to breach statistics were observed to be based primarily on the flow rate of the water rushing over the levee. This appears logical, as a higher velocity implies more shear stress, and more opportunity to surpass the soil's critical shear stress and cause erosion. Secondly, soil erodibility impacted the level of erosion as well. Within a single flow rate's time set, highly erodible soil failed first. The slope of the levee geometry had minimal impact

on times to breach, an observation that is somewhat surprising considering how important levee slope is in the design of levees, as it has an impact on levee seepage and levee stability. Different soils with different mixture percentage are also being considered for building the future levees. Therefore the results of the research would not be any more limited to only one kind of soil and by calibrating the computer model, normalized results can be obtained and thus the results can be extended for any kind of soils. To study the effect of seepage on over topping and erosion, levees with clayey core are currently being built. More centrifuge tests in higher  $g$ 's have been planned to be performed in future to investigate the effect of these new parameters on levees erosion.



Fig. 4. The levee with 87.5cm length and 39.5cm width failed in centrifuge test; water flow = 0.56 lit/min,  $g = 20$

Fig. 7 shows a visualization of the average times to breach of each experimental flow rate, levee slope, and soil erodibility. Each data point represents a single erosion simulation, and planes are colored to represent the points that were used to determine a single characteristic's average time to breach. For instance, in the left image, all data sets with a flow rate of 8 mL/s are represented by red, 11 mL/s by green, and 14 mL/s by blue data points. The bars on each axis represent the average time to breach of all data points of the corresponding color, and each image compares averages across a single characteristic. We can see that levee slope and erodibility have little effect on the times to breach, whereas flow rate has a major impact

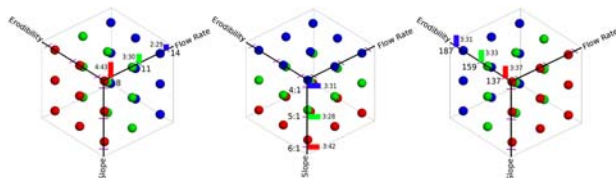


Fig. 7. A visualization of the average times to breach of each experimental flow rate, levee slope, and soil erodibility

#### 4. Conclusions

An investigation of various overtopping quantities dealing with levee erosion has been performed. Digital simulations have been presented to predict the time that it would take the levee to breach under different water flows. Additional centrifuge tests are planned. Since the breaching in centrifuge tests happens rapidly, some modification may be needed for the centrifuge tests; e.g.

using cameras with higher quality to capture better images and videos during and after the tests. It will help to observe and measure the exact breaching time during centrifuge tests; because even a few seconds in high " $g$ " tests represents a significant amount of prototype time. The following specific conclusions can be drawn from the study:

1. Higher water flow will lead to smaller  $t_{breach}$ . That is, in similar levees with different water flows, breaching would happen faster in the one which undergoes a higher water flow.
2. At higher water flows, most of the water will overtop the levee and the amount of water that seeps through the levee is negligible compared to overtopped water.
3. At smaller water flows (smaller than 0.4 lit/min), the amount of water that seeps through the soil is significant compared to the amount of water that overtops.
4. At small water flows, seepage plays a significant role on controlling the erosion. That is, long term seepage may eventually cause failure, but for short times it tends to reduce erosion.

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