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Modeling Crash Delays in a Route Choice Behavior Model for Two-Way Road Networks

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Abstract

Distributing demand in a transportation network is based on route choice behavior models. Generally, it is assumed that drivers use routes with minimum time. In real world, drivers may consider many factors other than travel times in congested networks especially in metropolitan or two way congested transportation networks. Travel safety is a factor that one may consider in his/her trip route choice. The main objective of this paper was to investigate influence of safety factors such as crash delays on drivers' route choice behaviors. Parameters that can cause to crash occurrences were specified and their impacts were modeled at macroscopic level using a simple statistical model. Then, an equilibrium based mathematical programming model for two way networks with symmetric link interactions was proposed. The model was tested for a simple network and results showed that how crash delays can impact on route choice behaviors.

Keywords**:** route choice, crash delay, two way networks

1. Introduction

Transportation networks are important infrastructures for societies. Urban developments and dispersing social and economic opportunities make it vital for planners to provide a sustainable transportation networks. Congestion, emissions, and high travel times are some of problems that cause decreasing in life quality. Therefore, it is imperative that planners use more accurate tools to analyze and predict transportation networks conditions. Traditionally, the most popular travel demand forecasting process extensively used by transportation modelers for decades are known as four-step models. Four step modeling is used to predict and evaluate transportation network conditions for future. Traffic assignment is the final step of transportation planning in which predicted trips are assigned to transportation networks. Route choice models play important role in traffic assignment process. Deterministic and stochastic user equilibrium models are two bases of route choice models. Deterministic user equilibrium (DUE) is based on Wardrop's law: The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route. In other words, all travelers selfishly make their route choices that result in a stable equilibrium traffic flow pattern such that there is no incentive for anyone to change his/her route [1]. In Stochastic User Equilibrium (SUE) models the perfect knowledge of users on travel times is relaxed: "At SUE, no motorists can improve his or her perceived travel time by unilaterally changing routes" [1, 2].

Many researchers have extended these two basic types of route choice models to capture uncertainties in both supply and demand aspects [3, 4]. To show the necessity of the subject, some experiments were implemented and results showed that travel time variability was either the most or second most important factor for most commuters [5]. Therefore, many studies have been conducted to incorporate travel time variability and users' risk behavior in route choice models. Bell and Cassir (2002) proposed a new equilibrium in respect to uncertainty in travel times. They showed that deterministic user equilibrium traffic assignment is equivalent to the mixed-strategy Nash equilibrium of an n-player, non-cooperative game and the mixed-strategy Nash equilibrium of this game describes a risk-averse user equilibrium traffic assignment [6]. Lo et al. (2006) proposed a route choice model based on travel time budget concept. They postulated that travelers acquire the variability of route travel times based on past experiences and factor such variability into their route choice consideration in the form of a travel time budget and all travelers want to minimize their travel time budgets. They formulated a multi-class mixed-equilibrium mathematical program to capture the route choice behaviors of travelers with heterogeneous risk aversions or requirements on punctual arrivals [7]. To extend stochastic route choice models, Mirchandani and Soroush (1987) proposed a generalized traffic equilibrium problem on stochastic networks (GTESP) that incorporates both probabilistic travel times and variable perceptions in the route choice decision process [8]. Siu and Lo (2006) formulated a stochastic equilibrium to address uncertainty in the actual travel time due to random link capacity degradations and perception variations in their travel time budget due to imperfect traffic information [9]. To address the effect of other parameters on drivers' route choice, Aashtiani and Iravani (1999) incorporated signalized and un-signalized delay to the deterministic traffic assignment. They proposed some delay functions based on HCM manual and they incorporated these functions to link delay function. They concluded that considering these delays will lead to more realistic results for planning purposes [10].

In reality drivers may consider many factors other than travel times in congested networks especially in metropolitan or two way congested transportation networks. Travel safety is a factor that one may consider in his/her trip route choice. For example, one may choose safer route between two paths with negligible in travel cost but with high different safety levels. Therefore, it is necessary to model drivers' route choice behaviors in a realistic manner. In transportation literature, many researches have been conducted to study on either transportation infrastructures safety [11, 12] or traffic safety [13, 14, 15]. However, none of them investigated the impacts of safety levels on route choice behaviors. The main objective of this paper is to model effects of crash delays on a route choice model. Specifically, parameters on crash occurrences will be explored and their impacts will be modeled at macroscopic level using a simple statistical model. Then, an equilibrium based mathematical programming model for two way networks considering user's perception errors are presented. The reminder of this paper is organized as follows. Section 2 presents methodology of incorporating crash delays on a route choice model. The mathematical model and solution algorithm are discussed in this section. Model results on a simple network are presented and discussed in section 3. Finally, summary of findings and recommendations for future studies are presented in section 4.

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2. Methodology

A traffic incident represents an event creating a temporary reduction in roadway capacity. These incidents depend on the severity can cause major or minor delays for travelers. The probability of occurring incidents in two-way transportation networks is higher comparing to one way networks. Therefore, it is necessary to consider that issue in route choice decision process. To clarify, consider a simple network presented in Fig 1. The network has two origin-destination pairs. There are two routes between origin 1 and destination 2. Route 1 is composed of link 1 which is a one-way link. Let's assume that it is a highway type link with a long length and high safety level. Route 2 is composed of link 2 which is a two-way link. Let's assume that it is a street type link with shorter length comparing to link 1. Now consider drivers who want to travel from node 1 to node 2. In classic route choice models, drivers only consider travel time to choose their route but in this case one may be also consider the risk that he/she may be counter.

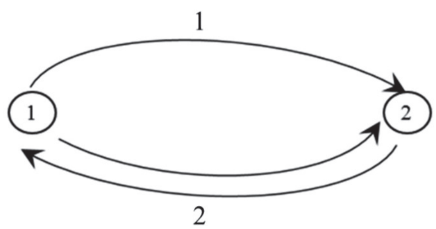


Fig. 1. Simple two-way network

When an incident occurs in a link, the capacity of that link would decrease during the incident and a queue would be built if traffic volume exceeds the capacity. The queue growing speed equals to difference between the rate of arrival and exited vehicles. Therefore, the maximum queue length on each link can be calculated as follow:

$$Q = (x_a - rC_a)T \tag{1}$$

where Q is the maximum queue length in vehicles unit, x_a is the traffic volume in link a in vehicles per time unit, r is the percentage of link capacity which would be available during an incident $0 < r < 1$, C_a is the capacity of the link a in vehicles per time unit, and T is the incident duration in time unit. Considering uniform distribution delays imposed to all users, the average length of queue can be estimated as half of maximum length. Hence, the total delay can be calculated as follows:

$$D_a = \frac{1}{2}QT = \frac{1}{2}(x_a - rC_a)T^2 \tag{2}$$

where D_a is the total delay that users experience for one incident in a particular link in time unit. To compute delay for each vehicle in link a, total delay can be divided to the total volume of link a:

$$D = \frac{1}{2x_a}(x_a - rC_a)T^2 \tag{3}$$

The following assumptions were considered to estimate the mean and variance of crash delays: 1. Incident duration follows gamma distribution with mean μ_a and variance σ_a^2 for each link and these parameters are deterministic, 2. The occurrence of incidents follow poisson process with parameter λ. λ_a is incident rate in unit length independent from volume for link a. Therefore, the expected number of incidents for the whole network can be calculated as follow:

$$\text{Expected number of incidents} = \sum_a \lambda_a x_a L_a \tag{4}$$

where L_a is the length of link a. Mean incident delay for each link can be obtained as follow:

$$\mu_a = E[D_a] = \frac{1}{2x_a}\lambda_a(x_a - rC_a)E[T^2] \tag{5}$$

where μ_α is mean of incident delay for link a. For random variable x we can write:

$$\sigma^2[x] = E[x^2] - (E[x])^2 \rightarrow E[T^2] = \sigma^2 + m^2 \tag{6}$$

substituting Eq. (6) in Eq. (5):

$$\mu = \frac{1}{2x_a}\lambda_a(x_a - rC_a)(\sigma^2 + m^2) \tag{7}$$

The unit of μ is in time per vehicle.mile. The mean delay for each vehicle can be computed as follow:

$$\begin{gathered}\mu_a = \frac{1}{2x_a}\lambda_a(x_a - rC_a)(\sigma_a^2 + m_a^2)x_a L \\ \Rightarrow \mu_a = \frac{1}{2}\lambda_a(x_a - rC_a)(\sigma_a^2 + m_a^2)L_a\end{gathered} \tag{8}$$

Eq. (8) presents mean delay that each vehicle expects due to incidents. The following equation can be obtained by rearranging Eq. (8):

$$\mu_a = \frac{1}{2}\lambda_a C_a(\frac{x_a}{C_a} - r)(\sigma_a^2 + m_a^2) \tag{9}$$

In this study it is assumed that traffic volumes in two directions can cause incidents. Aggregating link flows and capacities we have:

$$C = C_a + C_{a'} \tag{10}$$

$$X = x_a + x_{a'} \tag{11}$$

where a' stands for opposite link for link a. It should be noted that this formulation is applicable to congested network which their ratio of traffic volumes (sum of two opposite directions) to capacity (sum of two link capacity in each direction) is higher than the coefficient r. Expected crash delays can be computed by adding the following term to travel time function and it can be treated as a general cost function for links.

$$s_a(x_a, x_{a'}) = \frac{1}{2}\lambda_a C(\frac{X}{C} - r)(\sigma_a^2 + m_a^2) \tag{12}$$

where sa(xa,xa') is the delay term related to incidents. Link interactions can be either symmetric or asymmetric. In this study it is assumed that interactions are symmetric. This assumption implies that the effect of an additional flow unit along a particular link on travel time in the opposing direction equals the effect of an additional flow unit in the opposing direction on the travel time of the link under consideration. [1]. Hence, the generalized cost function for link a and the symmetry condition can be expressed mathematically as follow:

$$g_a(x_a, x_{a'}) = t_a(x_a, x_{a'}) + s_a(x_a, x_{a'}) \tag{13}$$

$$\frac{\partial g_a(x_a, x_{a'})}{\partial x_{a'}} = \frac{\partial g_{a'}(x_{a'}, x_a)}{\partial x_a} \tag{14}$$

It is an unreal assumption that drivers have perfect knowledge about the network condition and general costs. To relax that, a random error term is introduced to the generalized cost functions. It is assumed that this random error term follows Gumble distribution with following assumptions:

$$C_r^{ij} = c_r^{ij} + \varepsilon_r^{ij} \quad \forall r \in P_{ij}, i \in I, j \in J \tag{15}$$

$$E[C_r^{ij}] = c_r^{ij}, E[\varepsilon_r^{ij}] = 0 \tag{16}$$

where I and J are the set of origins and destinations respectively. Pij is the set of paths for a given origin i and destination j. εrij is the random error term for path r for a given origin i and destination j. Crij and crij are the stochastic and deterministic generalized cost of path r for origin i and destination j, respectively. Equivalent mathematical program to obtain equilibrium flows in the congested two way road networks considering safety levels and users' perception error is proposed as follow. At the equilibrium point no motorists can improve his or her perceived general cost by unilaterally changing routes.

$$\min_v \; z(x_a, x_{a'}) = \frac{1}{2}\sum_{a \in A}\left(\int_0^{x_a} g_a(\omega, x_{a'})\, \mathrm{d}\omega + \int_a^{x_a} g_a(\omega, 0)d\omega\right) + \frac{1}{\theta}\sum_{i \in I}\sum_{j \in J}\sum_{r \in P_{ij}} f_r^{ij} \ln f_r^{ij} \tag{17}$$

$$\text{s.t} \quad \sum_{r \in P_{ij}} f_r^{ij} = q_{ij} \qquad \forall\, i \in I \qquad j \in J \tag{18}$$

$$x_a = \sum_{i \in I}\sum_{j \in J}\sum_{r \in P_{ij}} f_r^{ij}\delta_{ar}^{ij} \quad \forall\, a \in A \tag{19}$$

$$f_r^{ij} \geq 0 \quad \forall\, r \in P_{ij} \qquad i \in I \quad j \in J \tag{20}$$

where ga( ) is the generalized cost function for link a, xa is the flow on link a, xa' is the flow on link a' (opposite direction of link a), frij is the flow on route k between origin i and destination j, qij is demand between origin i and destination j, θ is the dispersion parameter. The objective function has three terms. The first is the integral of the links' generalized performance function. The second term is the integral of the links' performance function and the third term captures users' perception errors. The first constraint assures that sum of path flows between an OD pair is equal to the demand of that OD pair. The second constraint is non negativity of path flows and the third constraint states that volume on the particular link is equal to the sum of path flows that is pass through that particular link.

2. 1. Solution algorithm

The Method of Successive Average (MSA) is applied to solve the mathematical model. This algorithm is relatively similar to the Frank-Wolf algorithm. The main difference between these two algorithms is related to step move size computation. Unlike to the Frank-Wolf algorithm, in the method of successive average move size is predetermined along the descent direction. Regularly, this step size is considered to be 1/n for each iteration where n is the number of iteration. To solve the mathematical program, the first step is to compute initial feasible solution. Stochastic loading can be applied with free flow travel generalized costs to obtain initial feasible solution. In the presence of loops in the network stochastic loading algorithms like STOCH algorithm can't be used. In this study it is assumed that all paths between an OD pair are reasonable provided that the path doesn't pass through each node many times. In other words, a reasonable path only can meet each node one time. The MSA algorithm can now be summarized as follows:

Step 0: Initialization. Perform a stochastic network loading based on a set of initial travel times {g0}. This generates a set of link flows {xa}. Set n = 1.
Step 1: Update. Set gan = ga(xan,xa'n)
Step 2: Direction finding. Perform a stochastic network loading procedure based on the current set of link general travel costs. This yields an auxiliary link flow pattern {yan}.
Step 3: Move. Find the new flow pattern by setting xan+1 = xan + (1/n)(yan – xan).
Step 4: Convergence criterion. If convergence is attained, stop. If not, set n = n + 1 and go to step 1. The convergence criterion is defined as below.

$$\frac{\sqrt{\sum_{a \in A}(x_a^{n+1} - x_a^n)^2}}{\sum_{a \in A} x_a^n} < \varepsilon \tag{21}$$

where ε is the accuracy of the algorithm. 0.00001 was used in this study.

3. Numerical example

In this section the proposed model is tested on a simple test network. The topology of the test network is depicted in Fig 2. This simple grid network has 6 nodes, 14 links and 2 OD pairs. One OD pair is 1-6 with a demand of 50 per minute and the other OD pair is 6-1 with a demand of 50 per minute. In this network pair links between nodes have interactions with each other.

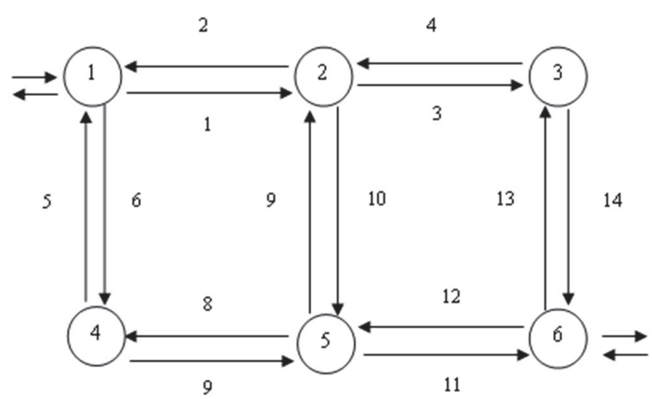


Fig. 2. Test network

The cost function given in the following equation is adopted to the test network:

$$g_a(x_a, x_{a'}) = t_a^0 + \alpha x_a + \beta x_{a'} + s(x_a, x_{a'}) \tag{22}$$

where t_a^0 is the free flow travel time on link a in minute unit and a' is the opposite link of link a. It is assumed that free flow travel time for all links is 3 minutes. Parameters and properties of the network are tabulated in Table 1. The sixth column shows the capacity of the links and the next columns show incident per unit, mean and standard deviation of incidents duration, respectively. It can be observed that links 3 and 4 have lower safety levels compared to other links. The dispersion parameter is assumed to be one and the parameter r was set to be 0.3 for all links. Table 2 presents the path number schemes.

Table 1: Link properties of the test network

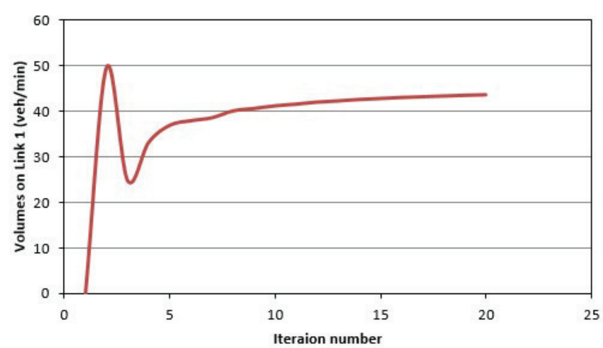
Link number	From	To	α	β	C (veh/min)	λ (crash/mile)	m (min)	σ^2 (min)
1	1	2	0.02	0.01	30	0.2	5	5
2	2	1	0.02	0.01	30	0.2	15	5
3	2	3	0.02	0.019	15	1	15	5
4	3	2	0.02	0.019	15	1	15	5
5	1	4	0.05	0.049	17	0.04	15	5
6	4	1	0.05	0.049	17	0.04	15	5
7	4	5	0.08	0.02	25	0.04	15	5
8	5	4	0.08	0.02	25	0.04	15	5
9	2	5	0.03	0.02	20	0.08	15	5
10	5	2	0.03	0.02	20	0.08	15	5
11	5	6	0.04	0.035	22	0.04	15	5
12	6	5	0.04	0.035	22	0.04	15	5
13	3	6	0.02	0.01	25	0.2	5	5
14	6	3	0.02	0.01	25	0.2	5	5

Table 2: Path number schemes

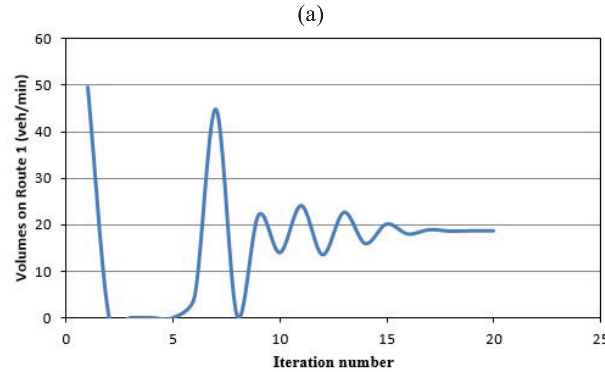
OD	Path number	Link number
1-6	1	1-3-13
	2	1-9-11
	3	5-7-11
	4	5-7-9-10-3-13
6-1	1	12-8-6
	2	14-4-2
	3	12-10-2
	4	14-4-9-8-6

4. Results

The presented mathematical model was implemented in MATLAB and assignment results were computed. Fig 3 presents the convergence of the algorithm for link 1 and path 1. It can be observed that the trajectory of convergence has zigzag pattern in initial iterations and the pattern is more visible for link flow trajectory. Results show that the algorithm converged to the equilibrium flows after about 17 iterations. Table 3 shows the assignment results. Link volumes, link costs and volume to capacity ratio is presented in this Table. According to the results, links 6, 8 and 12 have the highest volumes and volume to capacity ratio.
Table 4 presents path flows and path costs. It can be observed that for OD 1-6 paths 1, 2 and 3 were used and unlike to deterministic user equilibrium, used paths don't have equal costs but the majority of users recognize the path with minimum cost. For OD 6-1 only path 1 was used and it had considerably lower cost than other paths. This implies that in this case even if users have perception errors but they could recognize the paths with considerable lower costs.



(a)



(b)

Fig. 3. Trajectories of convergence (a). link1, (b) route 1

4. 1. Flow pattern comparisons for different dispersion parameter

To investigate the impacts of dispersion parameter on the flow patterns, model was run for different values of disp parameters and flows and related costs were computed. Table 5 shows the link flows, link costs and volume to capacity ratio for dispersion parameter 0.1, 1 and 10. The last column presents the assignment results when users have exact knowledge about network generalized costs. According to Table 5, link volumes change considerably especially in lower values of dispersion parameter. In transition from θ = 0.1 to θ = 1 high change in link flow patterns can be observed. Another conclusion that can be inferred from this Table is that when the dispersion parameter reaches to higher values, link flow patterns become more similar to deterministic assignment link flow patterns. It is an expected result because when the dispersion parameter becomes high value, it means that users' perception error is low and users have more sense about routes costs.

Table 3: Assignment results

Link number	From	To	Volume (veh/min)	Link Cost (min)	x/c Ratio
1	1	2	45.809	8.59	1.53
2	2	1	0.210	26.81	0.01
3	2	3	18.510	13.50	1.23
4	3	2	0.210	13.48	0.01
5	1	4	4.191	9.30	0.25
6	4	1	49.790	9.34	2.93
7	4	5	4.191	7.58	0.17
8	5	4	49.790	10.32	1.99
9	2	5	27.301	6.37	1.37
10	5	2	0.002	6.10	0.00
11	5	6	31.490	11.68	1.43
12	6	5	49.790	11.77	2.26
13	3	6	18.510	4.92	0.74
14	6	3	0.210	4.74	0.01

To better understanding the route choice behavior, Fig 4 is provided. This figure shows distribution of path cost with bar chart and the probability of choosing these paths with line for different dispersion parameter values. Note that routes 5, 6, 7 and 8 are the route numbers between OD 6-1. The figure implies that a path with lower cost has a higher chance to be chosen by travelers. For lower levels of the dispersion parameter, routes 2 and 3 of OD 1-6 have higher probability to be chosen but with growing in the parameter, the probability of path 1 and 2 increases. For OD 6-1 route 1 has the highest probability for all level of the parameter and the probability of choosing this route increases with growing in the dispersion parameter.

5. Conclusions and recommendations for future studies

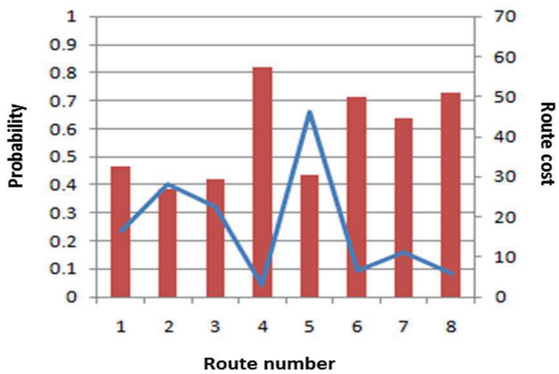
In this study, a route choice model considering crash delays and users' perception errors was proposed for two-way road networks with symmetric link interactions. The proposed model was performed on a simple grid network and various scenarios were analyzed and flow patterns were investigated due to changing in perception errors. For future research, this model can be improved by considering other parameters that influence on incident delays. In this study path enumeration was used in stochastic loading. Obviously, it is not applicable for real networks. Hence, this part of assignment can be improved by an efficient loading algorithm. Inability to account for overlapping and perception variance among routes are drawbacks of logit based stochastic models. This model can be further enhanced by considering these issues.

Table 4: Assignment results (route flows)

OD	Path number	Links	Path Flows (veh/min)	Path Cost (min)
1-6	1	1-3-13	18.72	27.01
	2	1-9-11	27.28	26.63
	3	5-7-11	4.00	28.55
	4	5-7-9-10-3-13	0	47.76
6-1	1	12-8-6	50.00	31.43
	2	14-4-2	0	45.03
	3	12-10-2	0	44.68
	4	14-4-9-8-6	0	44.25

Table 5: Assignment results (route flows)

		θ = 0.1		θ = 1		θ = 10		Deterministic	
OD	Path Num	Flow (veh/min)	Cost (min)	Flow (veh/min)	Cost (min)	Flow (veh/min)	Cost (min)	Flow (veh/min)	Cost (min)
1-6	1	11.74	32.47	18.72	27.01	18.79	27.58	0	27.69
	2	20.09	27.10	27.28	26.63	29.62	27.53	50	27.69
	3	16.12	29.30	4	28.55	1.59	27.83	0	27.69
	4	2.06	57.54	0	47.76	0	48.06	0	48.10
6-1	1	33.02	30.47	50	31.43	50	30.85	50	30.76
	2	4.70	49.97	0	45.03	0	47.19	0	47.56
	3	8.09	44.53	0	44.68	0	47.14	0	47.56
	4	4.19	51.10	0	44.25	0	44.26	0	44.27



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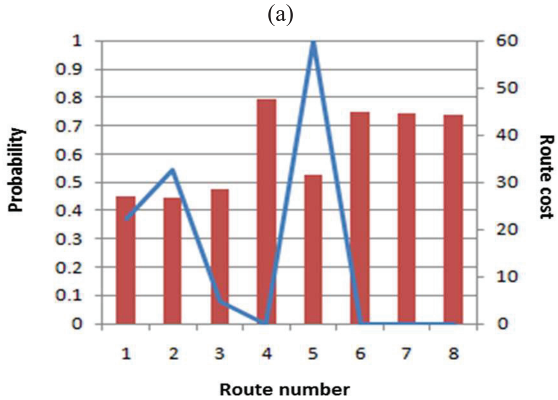

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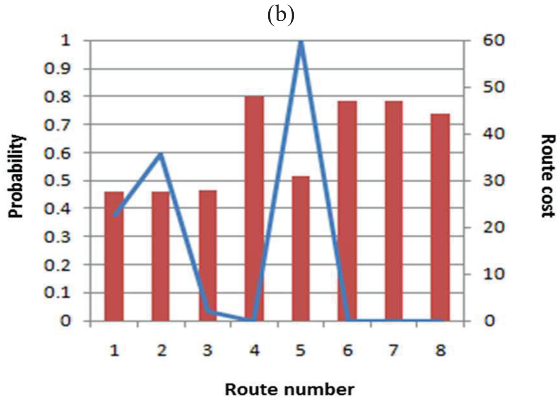

(c)

Fig. 4. Route costs and their probability (a). θ = 0.1, (b). θ = 1, (c). θ = 10

References

[1] Sheffi, Y. "Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods." Prentice-Hall, Incorporated, Englewood Cliffs, NJ, 1985.

[2] Daganzo, C. F., Sheffi, Y. "On Stochastic Models of Traffic Assignment." Transportation Science, vol. 11(3), pp. 253-274, 1977.

[3] Soltani-Sobh, A., Heaslip, K., El Khoury, J. "Estimation of road network reliability on resiliency: An uncertain based model." International Journal of Disaster Risk Reduction, vol. 14, pp. 536-544, 2015.

[4] Soltani-Sobh, A., Heaslip, K., Stevanovic, A., El Khoury, J., Song, Z. "Evaluation of transportation network reliability during unexpected events with multiple uncertainties." International Journal of Disaster Risk Reduction, In Press.

[5] Abdel-Aty, M., Kitamura, R., Jovanis, P. "Exploring route choice behavior using geographical information system-based alternative routes and hypothetical travel time information input," Transportation Research Record 1493, pp. 74-80, 1995.

[6] Bell, M. G. H., Cassir, C. "Risk-averse user equilibrium traffic assignment: an application of game theory," Transportation Research Part B, vol. 36 (8), pp. 671–681, 2002.

[7] Lo, H. K., Luo, X. W., Siu, B. W. Y. "Degradable transport network: travel time budget of travelers with heterogeneous risk aversion." Transportation Research Part B, vol. 40 (9), pp. 792–806, 2006.

[8] Mirchandani, P., Soroush, H. "Generalized traffic equilibrium with probabilistic travel times and perceptions." Transportation Science, vol. 21 (3), pp. 133–152, 1987.

[9] Siu, B. W. Y., Lo, H. K. "Doubly uncertain transport network: degradable link capacity and perception variations in traffic conditions." Transportation Research Record 1964, pp. 59–69, 2006.

[10] Aashtiani, H.Z., Iravani, H. "Use of intersection delay functions to improve reliability of traffic assignment model." Presented at the 14th Annual International EMME/2 Conference, Chicago, Illinois, 1999.

[11] Zolghadri, N., Halling, M., Barr, P. "Comparison of Wireless and Wired Structural System Identification." Structures Congress 2014, pp. 2839-2852, 2014. "doi: 10.1061/9780784413357.248"

[12] Zolghadri, N., Halling, M., Barr, P. "Effects of Temperature Variations on Structural Vibration Properties." Geotechnical and Structural Engineering Congress 2016, pp. 1032-1043, 2016. "doi: 10.1061/9780784479742.087"

[13] Khalilikhah, M., Heaslip, K. "Important environmental factors contributing to the temporary obstruction of the sign messages." Transportation Research Board 95th Annual Meeting, Washington, D.C., 2016.

[14] Khalilikhah, M., Heaslip, K. "GIS-based study of the impacts of air pollutants on traffic sign deterioration." Transportation Research Board 95th Annual Meeting, Washington, D.C., 2016.

[15] Baratian, F., Zhou, H. "Effects of photo enforcement cameras on intersection delays and driver behavior." Transportation Research Board 95th Annual Meeting, Washington, D.C., 2016.